LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2013**

MT 6604/MT 5500 – MECHANICS - II

Date : 13/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

PART – A

Answer ALL questions: (10 x 2 = 20 marks)

1. Define centre of gravity.
2. Define centre of mass of a rigid body.
3. State the principle of virtual work.
4. Define catenary.
5. A body moving with simple harmonic motion has an amplitude a and period T. Show that the velocity v at a distance x from mean position is given by $T^2 = 4\pi^2(a^2 - x^2)$.
6. Define central orbit.
7. Define simple pendulum.
8. Write down the p-r equation of a central orbit.
9. Write down the formula for moment of lnertia of i) a thin uniform rod ii) a rectangular lamina.
10. State D'Alembert's principle.

PART – B

Answer any FIVE questions: (5 x 8 = 40 marks)

11. Find the centre of gravity of a uniform hollow hemisphere of radius r.
12. Derive the intrinsic equation of the catenary.
13. Show that the composition of two simple harmonic motions of the same period along two perpendicular lines is an ellipse.
14. A regular hexagon is composed of six equal heavy rods freely jointed together and two opposite angles are connected by a string which is horizontal, one rod being in contact with a horizontal plane; at the middle point of the opposite rod a weight W' is placed. If W be the weight of each rod, show that tension in the string is $\frac{3W + W^1}{\sqrt{3}}$.
15. Derive the differential equation of a central orbit in the form $\frac{d^2u}{d\theta^2} + U = \frac{\overline{F}}{h^2u^2}$.
16. A particle describes the orbit $r = ae^{\theta\cot\alpha}$ under a central force, the pole being centre. Find the law of force.
17. State and prove the theorem of parallel axes.
18. Find the moment of inertia of a solid sphere.

PART – C

Answer any TWO questions: **(2 x 20 = 40 marks)**

19. a) Find the centre of gravity of a uniform solid right circular cone. (10)

b) Find the centre of gravity of the area enclosed by the parabolas $y^2 = ax$ and $x^2 = by$ ($a > 0, b > 0$). (10)

20. a) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contract. If θ and φ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\varphi = \frac{3}{8} + \tan\theta$. (10)

b) A telegraph wire, stretched between two points at a distance 'a' feet a part, sags n feet in the middle. Prove that the tension at the ends is approximately $W\left(\frac{a^2}{8n} + \frac{7}{6}n\right)$, where w is the weight per unit length of the wire. (10)

21. a) A particle moves in a SHM in a straight line. In the first second, after starting from rest, it travels a distance a and in the next second, it travels a distance b in the same direction. Prove that the amplitude of the motion is $\frac{2a^2}{3a-b}$. (10)

b) A particle describe a curve (for which s and ψ vanish simultaneously) with uniform speed v. If the acceleration at any point be $\frac{v^2c}{s^2+c^2}$, find the intrinsic equation of the curve. (10)

22. a) Define compound pendulum. Derive the equation of motion of a compound pendulum. Show that the point of suspension and point of oscillation of a simple equivalent pendulum is inter changeable. (10)

b) Find the M.I. of an elliptic lamina. (10)

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